5<sup>th</sup> Grade CCGPS Math *Unit 2: Numbers and Operations in Base Ten* Student Study Guide

*Dear Student:*

*Please review the Math standards below to prepare for the Georgia Milestones test. This will require that you take your interactive math notebook home each night to review your notes, handouts, and examples from class. Set aside enough study time each day prior to the assessment to ensure that you will do your best. Thanks!* ☺

**Unit 2 Common Core Math Standards:**

**MCC5.NBT.1** *Understand the place value system from thousandths to one million.*

*- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).*

| PLACE VALUE AND DECIMALS | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| millions | hundred thousands | ten thousands | thousands | hundreds | tens | ones | and | tenths | hundredths | thousandths | ten-thousandths | hundred-thousandths | millionths |
| | | | | | | 3 | . | 2 | 5 | | | | |
| | | | | | | 4 | . | 1 | 7 | 2 | | | |
| | | | | | 2 | 5 | . | 0 | 3 | | | | |
| | | | | | | 0 | . | 1 | 6 | 8 | | | |
| | | | | 1 | 3 | 2 | . | 7 | | | | | |

Note that **0.168** has the same value as **.168**. However, the zero in the ones place helps us remember that **0.168 is a number less than one.** From this point on, when writing a decimal that is less than one, we will always include a zero in the ones place.

| Phrase | Decimal |
|---|---|
| fifty-six hundredths | 0.56 |
| nine tenths | 0.9 |
| thirteen and four hundredths | 13.04 |
| twenty-five and eighty-one hundredths | 25.81 |
| nineteen and seventy-eight thousandths | 19.078 |

| Decimal | Phrase |
|---|---|
| 0.005 | five thousandths |
| 100.6 | one hundred and six tenths |
| 2.28 | two and twenty-eight hundredths |
| 71.062 | seventy-one and sixty-two thousandths |
| 3.0589 | three and five hundred eighty-nine ten-thousandths |

## Expanded Form

We can write the whole number 159 in expanded form as follows: 159 = (1 x 100) + (5 x 10) + (9 x 1). Decimals can also be written in expanded form. Expanded form is a way to write numbers by showing the value of each digit. This is shown in the example below.

Example 4:

| Decimal | | Expanded form |
|---|---|---|
| 4.12 | = | $(4 \times 1) + (1 \times \frac{1}{10}) + (2 \times \frac{1}{100})$ |
| 0.9 | = | $(0 \times 1) + (9 \times \frac{1}{10})$ |
| 9.735 | = | $(9 \times 1) + (7 \times \frac{1}{10}) + (3 \times \frac{1}{100}) + (5 \times \frac{1}{1000})$ |
| 1.0827 | = | $(1 \times 1) + (0 \times \frac{1}{10}) + (8 \times \frac{1}{100}) + (2 \times \frac{1}{1000}) + (7 \times \frac{1}{10000})$ |

*MCC5.NBT.3 Read, write and compare decimals to thousandths.*

*- Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.*

Decimal numbers are compared in the same way as other numbers: by comparing the different place values from left to right. We use the symbols <, > and = to compare decimals as shown below.

| Comparison | Meaning |
|---|---|
| 0.2 > 0.15 | 0.2 is greater than 0.15 |
| 0.15 < 0.2 | 0.15 is less than 0.2 |
| 0.2 = 0.2 | 0.2 is equal to 0.2 |
| 0.15 = 0.15 | 0.15 is equal to 0.15 |

When comparing two decimals, it is helpful to write one below the other. This is shown in the next example.

Example 2: Which is greater, 0.57 or 0.549?

Analysis: Let's compare these decimals using a place-value chart.

| ones | and | tenths | hundredths | thousandths |
|---|---|---|---|---|
| 0 | . | 5 | 7 | 0 |
| 0 | . | 5 | 4 | 9 |

Answer: 0.57 is greater than 0.549.

Notation: 0.57 > 0.549

As you can see in the example above, 0.57 has fewer decimal digits than 0.549. It is easier to compare two decimals when you have the same number of decimal digits, so an extra zero was written to the right of the digit 7 in the decimal 0.57. We are able to do this because 0.57 and 0.570 are equivalent decimals.

Example 4: Order these decimals from least to greatest: 3.87, 3.0875, 3.87502, 3.807

We have been asked to order four decimal numbers. Let's start by writing one decimal beneath the other in their original order.

3.87
3.0875
3.87502
3.807

Next, examine each decimal, writing one or more zeros to the right of the last digit, so that all decimals have the same number of decimal digits.

3.87000
3.08750
3.87502
3.80700

Now we can compare two decimals at a time. We will write a number in a circle next to each decimal to denote its order.

3.87000 ③
3.08750 ①
3.87502 ④
3.80700 ②

From least to greatest, we get: 3.08750, 3.80700, 3.87000, 3.87502

Answer:     Ordering these decimals from least to greatest we get: 3.0875, 3.807, 3.87, 3.87502

*MCC5.NBT.4*  *Use place value understanding to round decimals to any place.*

27.17469 rounded to the nearest whole number is 27
36.74691 rounded to the nearest whole number is 37
12.34690 rounded to the nearest tenth is 12.3
89.46917 rounded to the nearest tenth is 89.5
50.02139 rounded to the nearest hundredth is 50.02
72.63539 rounded to the nearest hundredth is 72.64
46.83531 rounded to the nearest thousandth is 46.835
9.63967 rounded to the nearest thousandth is 9.640

**Rules for rounding decimals.**
1. Retain the correct number of decimal places (e.g. 3 for thousandths, 0 for whole numbers)
2. If the next decimal place value is 5 or more, increase the value in the last retained decimal place by 1.

*MCC5.NBT.7. Add and subtract decimals to hundredths. Explain the relationship between concrete models, drawings, and written representations.*

**Procedure:**     **To add decimals, follow these steps:**

1. Line up all the decimal points in a column.
2. When needed, write one or more extra zeros to the right so that both decimals have the same number of decimal digits.
3. Start on the right, and add each column in turn. (Add digits in the same place-value position.)
4. If you need to carry, remember to add the tens digit of that column to the next column.
5. Place the decimal point in the sum.
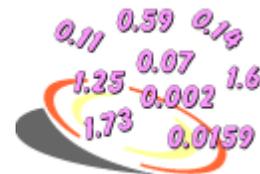
| Example 2: | Subtract: 8.06 **-** 8.019 |
|---|---|

**Step 1:** You must first line up the decimal points in a column.

```
  8.06
- 8.019
```

**Step 2:** The decimals in this problem do not have the same number of decimal digits. You can write an extra zero to the right of the last digit of the first decimal so that both decimals have the same number of decimal digits.

```
  8.060
- 8.019
```

**Step 3:** Start on the right, and subtract each column in turn. (Subtract digits in the same place-value position.)

```
      1
  8.060
- 8.019
      1
```

**Step 4:** If the digit you are subtracting is bigger than the digit you are subtracting from, you have to borrow a group of ten from the column to the left.

```
  5 1            5 1            5 1            5 1
  8.060          8.060          8.060          8.060
- 8.019        - 8.019        - 8.019        - 8.019
      1    →       41    →       041    →     0 041
```

**Step 5:** Be sure to place the decimal point in the difference.

```
  5 1            5 1            5 1            5 1
  8.060          8.060          8.060          8.060
- 8.019        - 8.019        - 8.019        - 8.019
      1    →       41    →       041    →     0.041
```

**Answer:** The difference is 0.041.

(cut and paste item here)

Most people in the world use a **base-10 system**. Each place in this system represents 10 times as much as the next place to the right. For example, 345 means 3 hundreds + 4 tens + 5 ones.
There are three types of blocks: *flats*, *rods*, and *cubes*.

Flat     Rod     Cube
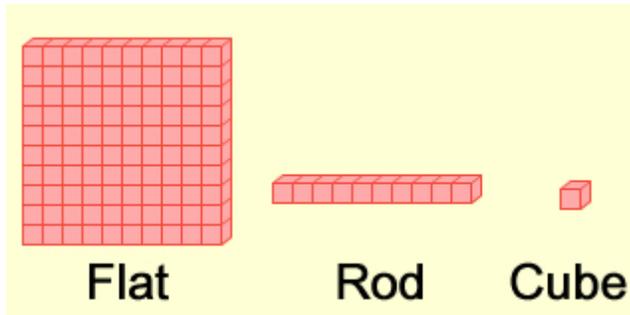
A <u>flat</u> is worth 1 whole.

A <u>rod</u> is worth one tenth (0.1).

A <u>cube</u> is worth one hundredth (0.01).

***Websites and Resource for <u>**Math Tutorials and Extra Practice**</u> (aligned to the 5th Grade Common Core State Standards):

http://www.helpingwithmath.com/by_grade/gr5_cc_skills.htm

http://www.k-5mathteachingresources.com/5th-grade-number-activities.html

http://www.khanacademy.org/

http://nsdl.org/commcore/math?id=5

https://sites.google.com/a/norman.k12.ok.us/mr-wolfe-s-math-interactive-whiteboard/5th-grade

http://pinterest.com/teaching4real/5th-grade-common-core/

http://www.mathscore.com/math/standards/Common%20Core/5th%20Grade/

http://www.ixl.com/math/standards/common-core/grade-5

http://intermediateelem.wikispaces.com/Fifth+Grade+Math+Resource+Backup

http://mail.clevelandcountyschools.org/~ccselem/?OpenItemURL=S07BB59E1

Dutchtown Elementary School
*"We are a family of learners"*

## 5<sup>th</sup> Grade Unit 3 CCGPS Math Study Guide: *Multiplying and Dividing with Decimals*

*Name*_____ *Date*_____

*Dear Student,*

*Please set aside enough time each night to review the following math standards to prepare yourself for the Georgia Milestones test in April. Remember to review all notes and handouts from your interactive math notebook as well as any problems missed on graded classwork and other formative assessments throughout the unit.*

Unit 3 CCGPS Math Standards:

**MCC5.NBT.2** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

**MCC5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Students should be able to demonstrate the following competencies:
• understand place value
• use whole number exponents to denote powers of 10
• compare decimals
• model multiplication and division of decimals
• multiply and divide decimals by powers of 10
• use estimation when multiplying and dividing decimals
• multiply and divide decimals with fluency
• determine relationship between quantities algebraically
• recognize student errors in multiplication and division of decimals
• use decimals to solve problems

## Patterns and Exponents:

The numbers 1, 2, 4, 8, 16, 32, 64, 128, 256, ... form a pattern. What is the rule for this pattern? (multiply by 2)

This list of numbers results from finding <u>powers</u> of 2 in sequence. Look at the table below and you will see several patterns.

| Exponential Form | Factor Form | Standard Form |
|---|---|---|
| $2^0 =$ | Any number (except 0) raised to the zero power is always equal to 1. | 1 |
| $2^1 =$ | Any number raised to the first power is always equal to itself. | 2 |
| $2^2 =$ | 2 x 2 = | 4 |
| $2^3 =$ | 2 x 2 x 2 = | 8 |
| $2^4 =$ | 2 x 2 x 2 x 2 = | 16 |
| $2^5 =$ | 2 x 2 x 2 x 2 x 2 = | 32 |

| $2^6 =$ | 2 x 2 x 2 x 2 x 2 x 2 = | 64 |
|---|---|---|
| $2^7 =$ | 2 x 2 x 2 x 2 x 2 x 2 x 2 = | 128 |
| $2^8 =$ | 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 = | 256 |

Can you predict the next two numbers in the list after 256? (512 and 1,024)

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Example 1:    Rewrite the numbers 1, 3, 9, 27, 81, 243, ... as a powers of 3.

Solution:

| Exponential Form | Standard Form |
|---|---|
| $3^0 =$ | 1 |
| $3^1 =$ | 3 |
| $3^2 =$ | 9 |
| $3^3 =$ | 27 |
| $3^4 =$ | 81 |
| $3^5 =$ | 243 |

Can you predict the next two numbers in the list after 243?  (729 and 2, 187)

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Example 2: If $7^3 = 343$, then find $7^4$ with only one multiplication.

Solution:    $7^4 = 7^3$ times 7

$7^4 = 343 \times 7$

$7^4 = 2,401$

---

Example 3: If $4^5 = 1,024$, then find $4^6$ with only one multiplication.

Solution:    $4^6 = 4^5$ times 4

$4^6 = 1,024 \times 4$

$4^6 = 4,096$

---

Example 4: If $10^0 = 1$, and $10^1 = 10$, and $10^2 = 100$, and $10^3 = 1,000$, then predict the values of $10^6$ and $10^8$ in standard form.

Solution:

| Exponential Form | Standard Form | Description |
|---|---|---|
| $10^0 =$ | 1 | The exponent is 0; the number 1 has no zeros. |
| $10^1 =$ | 10 | The exponent is 1; the number 10 has 1 zero. |
| $10^2 =$ | 100 | The exponent is 2; the number 100 has 2 zeros. |
| $10^3 =$ | 1,000 | The exponent is 3; the number 1,000 has 3 zeros. |
| $10^6 =$ | 1,000,000 | The exponent is 6; the number 1,000,000 has 6 zeros. |

| | | |
|---|---|---|
| $10^8 =$ | 100,000,000 | The exponent is 8; the number 100,000,000 has 8 zeros. |

Summary: When you find powers of a number in sequence, the resulting list of products forms a pattern. By examining this pattern, we can predict the next product in the list. Given the standard form of a number raised to the $n^{th}$ power, we can find the standard form of that number raised to the n+1 power with a single multiplication. When you find powers of 10 in sequence, a pattern of zeros is formed in the resulting list of products.

## Multiplying Decimals:

Problem:  Two students multiplied 0.2 by 0.4. Student 1 found a product of 0.8 and Student 2 found a product of 0.08. Which student had the correct answer? Explain.

| Student 1 | Student 2 |
|---|---|
| 0.2 | 0.2 |
| x 0.4 | x 0.4 |
| 0.8 | 0.08 |

Analysis:  Let's convert each decimal to a fraction to help us solve this problem.

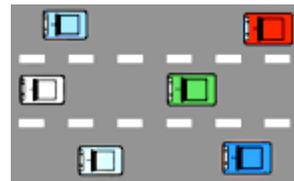|  | Factors | | Product |
|---|---|---|---|
| Fractions: | $\frac{2}{10}$ x | $\frac{4}{10}$ = | $\frac{8}{100}$ |
| Decimals: | 0.2 x | 0.4 = | 0.08 |

If we multiply two tenths by four tenths, we get a product of eight hundredths.

Answer:  Student 2 is correct since 0.2 x 0.4 = 0.08.

In the problem above, the product 0.08 is smaller than each individual factor. This is because you are finding a part of a part: 0.2 of 0.4 is 0.08. Thus, when you multiply two decimal factors, where each is less than one, the product will be less than each individual factor. Let's look at some more examples of multiplying decimals.

Example 1:  Marina's car gets 44.8 miles per gallon on the highway. If her fuel tank holds 15.4 gallons, then how far can she travel on one full tank of gas?

Analysis:  We can multiply 44.8 by 15.4 to solve this problem.

Step 1:  **Estimate the product.**

Round both factors down.
44.8 → 40
x 15.4 → x 10
400

Round both factors up.
44.8 → 50
x 15.4 → x 20
1000

The product of 44.8 and 15.4 ranges from 400 to 1000.

Step 2:  **Multiply to find the product.**

Multiply these decimals as if they were whole

Compensate by placing the decimal point in the

numbers. Ignore the decimal point.                         product.

```
        44.8                              44.8  ← 1 decimal digit
    x   15.4                          x   15.4  ← 1 decimal digit
    ─────────                         ─────────
       17 92                             17 92
      224 00                            224 00
    ─────────                         ─────────
      689 92                            689.92  ← 2 decimal digits
```

Step 3:      **Compare your estimate with your product to verify that your answer makes sense.**

             Our answer of 689.92 makes sense since the product of 44.8 and 15.4 ranges from 400 to 1,000.

Answer:      Marina can travel 689.92 miles with one full tank of gas.


When multiplying decimals, placement of the decimal point is very important. Since there is one decimal digit in each factor, there must be two decimal digits in the product. This is because tenths x tenths = hundredths. Estimating the product lets us verify that the placement of the decimal point is correct, and that we have a reasonable answer. For example, if your product was 68.992 miles, then you would know that you made a multiplication error after comparing it with your estimate. Let's look at some more examples of multiplying decimals.


Example 2:   Multiply: 3.54 x 1.8

Analysis:    There are two decimal digits in the factor 3.54 and 1 decimal digit in the factor 1.8.

Step 1:      **Estimate the product.**

```
       3.54  →      3
   x   1.8   →   x  2
   ────────      ─────
                     6
```

Step 2:      **Multiply to find the product.**

             Multiply these decimals as if they were whole          Compensate by placing the decimal point in the
             numbers. Ignore the decimal point.                     product.

```
        3.54                             3.54  ← 2 decimal digits
    x   1.8                          x   1.8   ← 1 decimal digit
    ─────────                        ─────────
       2 832                            2 832
       3 540                            3 540
    ─────────                        ─────────
       6 372                            6.372  ← 3 decimal digits
```

Step 3:      **Compare your estimate with your product to verify that your answer makes sense.**

             Our product of 6.372 makes sense since it is close to our estimate of 6.

Answer:      The product of 3.54 and 1.8 is 6.372


In Example 2, there are two decimal digits in the factor 3.54 and one decimal digit in the factor 1.8. Therefore, there must be three decimal digits in the product. Perhaps you are wondering why this is so. When we ignored the decimal point in Step 2, we really moved it two places to the right for the first factor (3.54 x 100 = 354.) and one place to the right for the second factor (1.8 x 10 = 18.). We need to compensate to get the right answer. To do this, we must add up the total number of places the decimal point was moved to the right. Then, starting from the right of the last digit in the product, we must move the decimal point the same number of places to the left. In short, since we multiplied by 10 to the third power, we must compensate by dividing by 10 to the third power.

# Dividing Decimals by Whole Numbers:

Example 1:   The Lachance family drove cross country on a 4,615.8 mile trip in 49 days.
Find the average number of miles driven per day.

Analysis:   We need to divide 4,615.8 by 49 to solve this problem.

Step 1:   **Estimate the quotient using compatible numbers.**

$$\overset{\text{quotient}}{\text{divisor}\overline{)\text{dividend}}} \qquad 49\overline{)4,615.8} \rightarrow 50\overline{)4,500}\,^{90}$$

Step 2:   **Use long division to find the quotient.**

**Decide where to place the first digit of the quotient.**

$$\overset{\downarrow}{49\overline{)4,615.8}}$$  49 does not go into 4 and 49 does not go into 46. Therefore, we must start with 461 divided by 49. The first digit of the quotient will be in the tens place.

**Round to estimate the quotient digit.**

$$49\overline{)4,615.8}\,^{9}$$  Think: $50\overline{)461}$ or $5\overline{)45}$  Try 9.

**Multiply, subtract and compare.**

$$\begin{array}{r} 9 \\ 49\overline{)4,615.8} \\ \underline{4\ 41} \\ 20 \end{array}$$

Multiply the quotient digit by the divisor: $9 \times 49 = 441$
Subtract: $461 - 441 = 20$
Compare: Is the difference less than the divisor? Yes: $20 < 49$

**Bring down the next digit from the dividend. Continue dividing.**

$$\begin{array}{r} 9 \\ 49\overline{)4,615.8} \\ \underline{4\ 41} \\ 205 \end{array} \qquad \begin{array}{r} 94 \\ 49\overline{)4,615.8} \\ \underline{4\ 41} \\ 205 \\ \underline{196} \\ 9 \end{array} \qquad \begin{array}{r} 94 \\ 49\overline{)4,615.8} \\ \underline{4\ 41} \\ 205 \\ \underline{196} \\ 9\ 8 \end{array} \qquad \begin{array}{r} 94\ 2 \\ 49\overline{)4,615.8} \\ \underline{4\ 41} \\ 205 \\ \underline{196} \\ 9\ 8 \\ \underline{9\ 8} \\ 0 \end{array}$$

Bring down the 5.          4 x 49 = 196          Bring down the 8.          2 x 49 = 98

**Place the decimal point in the quotient.**

```
            94.2
    49)4,615.8
       4 41
        205
        196
          9 8
          9 8
            0
```

**Check your answer: Multiply the divisor by the quotient to see if you get the dividend.**

```
          94.2
    x       49
         847 8
       3 768 0
       4,615.8
```

Step 3:   **Compare your estimate with your quotient to verify that your answer makes sense.**

Our quotient of 94.2 makes sense since it is close to our estimate of 90.

Answer:   The average number of miles driven per day was 94.2.

Estimating the quotient lets us verify that the placement of the decimal point is correct, and that we have a reasonable answer. For example, if our estimate was 90 and our quotient was 9.42, then we would know that we made a division error.

## Dividing Decimals by Decimals:

Example 1:   0.8)9.6

Analysis:   The divisor is 0.8. To make it a whole number, we will multiply both the dividend and the divisor by 10.

| **Multiply the divisor by a power of 10 to make it a whole number.** | **Multiply the dividend by the same power of 10. Place the decimal point in the quotient.** | **Divide the dividend by the whole-number divisor to find the quotient.** |
| --- | --- | --- |
| 08.)9.6 | 08.)96. | 12. 8)96. 8 16 16 0 |

Answer:   The quotient of 9.6 and 0.8 is 12.

In Example 1, we changed the divisor to a whole number before performing the division. To do this, **we multiplied both the divisor and the dividend by the same power of 10**. Note that the quotient of 9.6 and 0.8 is the same as the quotient of 96 and 8. Let's look at why this is possible:

Both the divisor and dividend were multiplied by 10: $\frac{9.6 \times 10}{0.8 \times 10}$. Since $\frac{10}{10} = 1$, this is the same as multiplying by 1.

Thus, the quotient of 9.6 and 0.8 and the quotient of 96 and 8 are both 12. Let's look at another example.

Example 2: 0.35⟌13.93

Analysis: The divisor is 0.35. To make it a whole number, we will multiply both the dividend and the divisor by 100.

| **Multiply the divisor by a power of 10 to make it a whole number.** | **Multiply the dividend by the same power of 10. Place the decimal point in the quotient.** | **Divide the dividend by the whole-number divisor to find the quotient.** |
|---|---|---|
| 035.⟌13.93 | 035.⟌1,393. | 39.8<br>35⟌1,393.0<br>1 05<br>343<br>315<br>28 0<br>28 0<br>0 |

Answer: The quotient of 13.93 and 0.35 is 39.8

Note that in Example 1, the quotient is a whole number (12), and in Example 2, the quotient is a decimal (39.8).

Example 3: 0.009⟌5.4

Analysis: The divisor is 0.009. To make it a whole number, we will multiply both the dividend and the divisor by 1,000.

| **Multiply the divisor by a power of 10 to make it a whole number.** | **Multiply the dividend by the same power of 10. Place the decimal point in the quotient.** | **Divide the dividend by the whole-number divisor to find the quotient.** |
|---|---|---|
| 0009.⟌5.4 | 0009.⟌5,400. | 600.<br>9⟌5,400.<br>5 4<br>000 |

Answer: The quotient of 5.4 and 0.009 is 600.

Example 4: Divide, then round the quotient to the nearest tenth: 3.06⟌201.4

Analysis: The divisor is 3.06. To make it a whole number, we will multiply both the dividend and the divisor by 100. After dividing, we will round the quotient to the nearest tenth.

| **Multiply the divisor by a power of 10 to make it a whole number.** | **Multiply the dividend by the same power of 10. Place the decimal point in the quotient.** | **Divide the dividend by the whole-number divisor to find the quotient.** |
|---|---|---|

```
                                                            65.81
  306.)201.4          306.)20,140.         306.)20,140.00
                                                            18 36
                                                             1 780
                                                             1 530
                                                               250 0
                                                               244.8
                                                                 5 20
                                                                 3 06
                                                                 2 14
```

Answer:    Rounded to the nearest tenth, the quotient of 201.4 and 3.06 is 65.8.

## Solving Decimal Word Problems:

Example 1:  School lunches cost $14.50 per week. About how much would 15.5 weeks of lunches cost?

Analysis:   We need to estimate the product of $14.50 and 15.5. To do this, we will round one factor up and one factor down.

Estimate:
```
  $14.50  →      $10
x    15.5  →   x   20
                  $200
```

Answer:    The cost of 15.5 weeks of school lunches would be about $200.

---

Example 2:  A student earns $11.75 per hour for gardening. If she worked 21 hours this month, then how much did she earn?

Analysis:   To solve this problem, we will multiply $11.75 by 21.

Multiply:
```
  $11.75
x     21
   11 75
  235 00
 $246.75
```
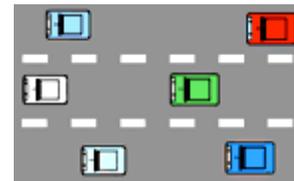
Answer:    The student will earn $246.75 for gardening this month.

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Example 3:  Rick's car gets 29.7 miles per gallon on the highway. If his fuel tank holds 10.45 gallons, then how far can he travel on one full tank of gas?

Analysis:   To solve this problem, we will multiply 29.7 by 10.45

Multiply:
```
   10.45
x    29.7
    7 315
   94 050
  209 000
  310.365
```

Answer:    Rick can travel 310.365 miles with one full tank of gas.

Example 4:   A member of the school track team ran for a total of 179.3 miles in practice over 61.5 days. About how many miles did he average per day?

Analysis:    We need to estimate the quotient of 179.3 and 61.5.

Estimate:

$$61.5 \overline{)179.3} \quad \rightarrow \quad 60 \overline{)180}^{\,3}$$

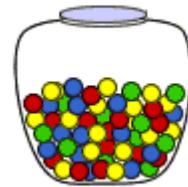Answer:      He averaged about 3 miles per day.

---

Example 5:   A store owner has 7.11 lbs. of candy. If she puts the candy into 9 jars, how much candy will each jar contain?

Analysis:    We will divide 7.11 lbs. by 9 to solve this problem.

Divide:

$$
\begin{array}{r}
0.79 \\
9 \overline{)7.11} \\
\underline{6\ 3} \\
81 \\
\underline{81} \\
0
\end{array}
$$

Answer:      Each jar will contain 0.79 lbs. of candy.

## Links for Online Practice (includes instant answer verification):

http://www.mathgoodies.com/lessons/decimals/practice_unit12.html  (spiral review of Math Unit 2)

http://www.mathgoodies.com/lessons/decimals_part2/practice_unit13.html

# 5th Grade CCGPS Math *Unit 4: Adding, Subtracting, Multiplying, and Dividing Fractions*
# Student Study Guide

**Dutchtown Elementary School**
*"We are a family of learners"*

Student: _____Homeroom Teacher: _____

*Dear Student:*

*Please review the Math standards below to prepare for the Georgia Milestones test in April.  This will require that you take your interactive math notebook home each night to review your notes, handouts, and examples from class. Set aside enough study time each day prior to the assessment to ensure that you will do your best. Thanks!* ☺

## Unit 4 Common Core Math Standards:

**MCC5.NF.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
*For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

**MCC5.NF.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

**MCC5.NF.3** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.  *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size*
*3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
**MCC5.NF.4** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
**a.** Interpret the product ($a/b$) × $q$ as a parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a × q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*
**b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
**MCC5.NF.5** Interpret multiplication as scaling (resizing), by:
**a.** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
**b.** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence
$a/b = (n × a)/(n × b)$ to the effect of multiplying $a/b$ by 1.
**MCC5.NF.6** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
**MCC5.NF.7** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.**1** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context

for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.

***Note to Parents***: Since this is our first year teaching the CCGPS curriculum, we are required to teach new standards that were not addressed in the previous 4th grade GPS curriculum for the Year 1 implementation.

Pre-requisite 4th Grade CCGPS Fraction Concepts taught prior to the beginning of the unit:

**Equivalent Fractions**

What do the fractions in example 1 have in common?
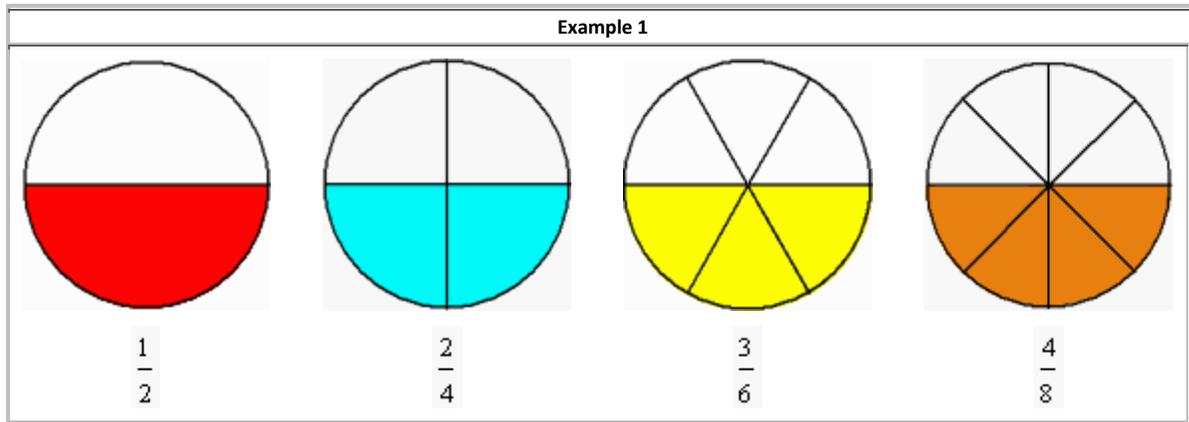
| **Example 1** |
| :---: |



$$\frac{1}{2} \qquad \frac{2}{4} \qquad \frac{3}{6} \qquad \frac{4}{8}$$
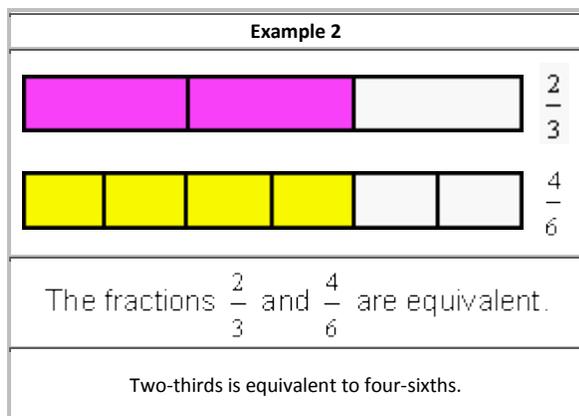
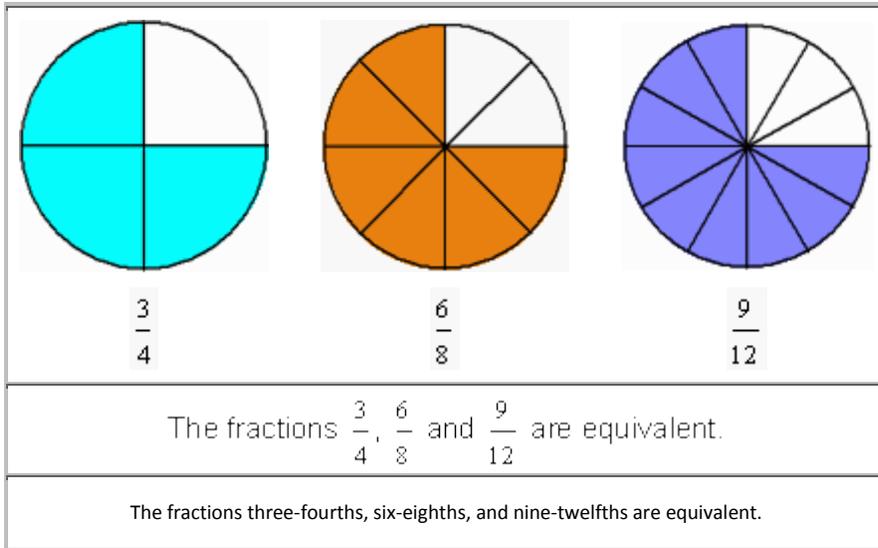Each fraction in example 1 represents the same number. These fractions are *equivalent*.

**Definition: Equivalent fractions** are different fractions that name the same number.

The fractions $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ and $\frac{4}{8}$ are equivalent since each represents the same number.

Let's look at some more examples of equivalent fractions.

| **Example 2** |
| :---: |



$$\frac{2}{3}$$

$$\frac{4}{6}$$

The fractions $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent.

Two-thirds is equivalent to four-sixths.

| **Example 3** |
| :---: |

The fractions $\frac{3}{4}$, $\frac{6}{8}$ and $\frac{9}{12}$ are equivalent.

The fractions three-fourths, six-eighths, and nine-twelfths are equivalent.

What would happen if we did not have shapes such as circles and rectangles to refer to? Look at example 4 below.

| Example 4 |
|---|
| Are the fractions $\frac{2}{3}$, $\frac{6}{9}$, and $\frac{8}{12}$ equivalent? Explain why or why not. |

We need an arithmetic method for finding equivalent fractions.

**Procedure:  To find equivalent fractions,** multiply the numerator AND denominator by the same nonzero <u>whole number</u>.

This procedure is used to solve example 4.

| Example 4 |
|---|
| Are the fractions $\frac{2}{3}$, $\frac{6}{9}$, and $\frac{8}{12}$ equivalent? Explain why or why not. |
| Part A:  $\dfrac{2}{3} = \dfrac{2 \times 3}{3 \times 3} = \dfrac{6}{9}$ <br><br> Part B:  $\dfrac{2}{3} = \dfrac{2 \times 4}{3 \times 4} = \dfrac{8}{12}$ |
| Yes, $\frac{2}{3}$, $\frac{6}{9}$, and $\frac{8}{12}$ are equivalent, since the numerator and denominator of each fraction was multiplied by the same nonzero number. |

**Simplifying Fractions**

You can multiply the numerator and the denominator of a fraction by any nonzero whole number, as long as you multiply both by the *same* whole number!

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We want to write fractions using
the smallest numbers possible.

</div>

For example, look at $\dfrac{100}{200}$

Can you think of a better way to write this using smaller numbers? Let's see...

What if we divided the numerator and denominator by 100?

$$\frac{100 \div 100}{200 \div 100} = \frac{1}{2}$$

Oh, yeah... This is a lot better!

Here are the two rules:

<div style="border: 2px solid blue;">

1) You can divide the numerator and the denominator by any number... as long as you use the same number for both! For example, you can't divide the numerator by 3 and the denominator by 5.

2) You need to get the numerator and denominator as small as possible.

</div>

**Harcourt Simplifying Fractions Video Lesson**: http://www.harcourtschool.com/activity/show_me/e622.htm

**Comparing Fractions**

**Example 1:** Drake rode his bike for three-fourths of a mile and Josh rode his bike for one-fourth of a mile. Which boy rode his bike farther?
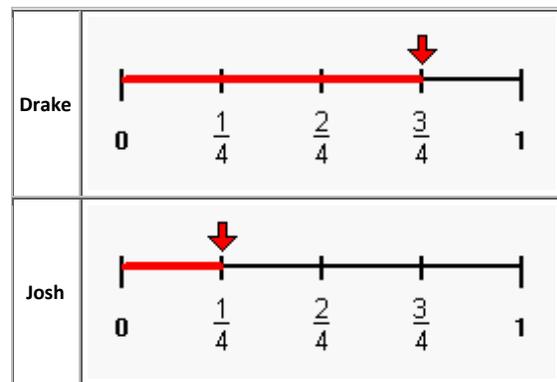
**Analysis:**

$\dfrac{3}{4}$ ? $\dfrac{1}{4}$



These fractions have like denominators, so we can compare the numerators.

**Solution:**

$\dfrac{3}{4}$ > $\dfrac{1}{4}$

Since three is greater than one, three-fourths is greater than one-fourth. Therefore, Drake rode his bike farther.

When comparing two fractions with like denominators, the larger fraction is the one with the greater numerator. Let's look at some

more examples of comparing fractions with like denominators.

**Example 2:** Compare the fractions given below using the symbols <, > or =.

a) $\dfrac{1}{3}$ ? $\dfrac{2}{3}$      $\dfrac{1}{3} < \dfrac{2}{3}$

b) $\dfrac{3}{2}$ ? $\dfrac{1}{2}$      $\dfrac{3}{2} > \dfrac{1}{2}$

c) $\dfrac{3}{4}$ ? $\dfrac{7}{4}$      $\dfrac{3}{4} < \dfrac{7}{4}$

d) $\dfrac{6}{6}$ ? $\dfrac{5}{6}$      $\dfrac{6}{6} > \dfrac{5}{6}$

e) $\dfrac{4}{3}$ ? $\dfrac{5}{3}$      $\dfrac{4}{3} < \dfrac{5}{3}$

f) $\dfrac{5}{5}$ ? $\dfrac{5}{5}$      $\dfrac{5}{5} = \dfrac{5}{5}$

**Example 3:** Josephine ate three-fourths of a pie and Penelope ate two-thirds of a pie. If both pies are the same size, then which girl ate more pie?

**Analysis:**

$$\dfrac{3}{4} \quad ? \quad \dfrac{2}{3}$$

These fractions have unlike denominators (and unlike numerators). It would be easier to compare them if they had like denominators. **We need to convert these fractions to equivalent fractions with a common denominator in order to compare them more easily.**

**Josephine:** $\dfrac{3}{4} = \dfrac{n}{12}$     $\dfrac{3}{4} = \dfrac{3 \times 3}{4 \times 3} = \dfrac{9}{12}$

**Penelope:** $\dfrac{2}{3} = \dfrac{n}{12}$     $\dfrac{2}{3} = \dfrac{2 \times 4}{3 \times 4} = \dfrac{8}{12}$

**Solution:**

$$\dfrac{3}{4} > \dfrac{2}{3} \text{ since } \dfrac{9}{12} > \dfrac{8}{12}$$

Since nine-twelfths is greater than eight-twelfths, three-fourths is greater than two-thirds. Therefore, Josephine ate more pie.

The example above works out nicely! *But how did we know to use 12 as our common denominator?* It turns out that the least common denominator is the best choice for comparing fractions.

**Definition:** The **least common denominator** (LCD) of two or more non-zero denominators is the smallest whole number that is divisible by

each of the denominators.

To find the least common denominator (LCD) of two fractions, find the <u>least common multiple</u> (LCM) of their denominators.

Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39,...
Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48,...
Common multiples of 3 and 4 are 12, 24 and 36.
The *least* common multiple of 3 and 4 is 12.
LCM = 12 ✔

Remember that "**...**" at the end of each list of multiples indicates that the list goes on forever. Revisiting example 3, we found that the least common multiple of 3 and 4 is 12. Therefore, the least common denominator of two-thirds and three-fourths is 12. We then converted each fraction into an equivalent fraction with a denominator of 12, so that we could compare them.

**Josephine:** $\dfrac{3}{4} = \dfrac{n}{12}$   $\dfrac{3}{4} = \dfrac{3 \times 3}{4 \times 3} = \dfrac{9}{12}$

**Penelope:** $\dfrac{2}{3} = \dfrac{n}{12}$   $\dfrac{2}{3} = \dfrac{2 \times 4}{3 \times 4} = \dfrac{8}{12}$

**Procedure:** To compare fractions with unlike denominators, follow these steps:

1. Use the LCD to write equivalent fractions with a common denominator.
2. Compare the numerators: The larger fraction is the one with the greater numerator.

**Ordering Fractions**

**Example 1:**   An 8-ounce cup of milk was served to each of three children. Lisa drank 7 ounces of milk. Her sister Angie drank 3 ounces, and her brother Mark drank 5 ounces. What part of the cup did each child drink? Who drank the smallest part of the cup? Who drank the largest part of the cup? Who fell in the middle?
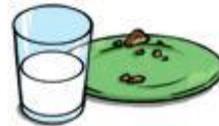
**Analysis:**   Write the part of the cup that each child drank as a fraction, and then order them from least to greatest.

| Child | Milk Drank | Fraction |
|-------|-----------|----------|
| Lisa  | 7 oz.     | $\dfrac{7}{8}$ |
| Angie | 3 oz.     | $\dfrac{3}{8}$ |
| Mark  | 5 oz.     | $\dfrac{5}{8}$ |

| Child | Milk Drank | Fraction | Order |
|-------|-----------|----------|-------|
| Lisa  | 7 oz.     | $\dfrac{7}{8}$ | ③ |
| Angie | 3 oz.     | $\dfrac{3}{8}$ | ① |
| Mark  | 5 oz.     | $\dfrac{5}{8}$ | ② |

$\dfrac{3}{8} < \dfrac{5}{8}$ and $\dfrac{5}{8} < \dfrac{7}{8}$

$\dfrac{3}{8} < \dfrac{5}{8} < \dfrac{7}{8}$

**Solution:**     Angie drank the smallest part of the cup. Lisa drank the largest part of the cup. Mark fell in the middle.

We were able to order these fractions from least to greatest because they have like denominators.

**To order fractions with like denominators, look at the numerators and compare them two at a time. It is helpful to write a number in a circle next to each fraction to compare them more easily.**

Let's look at another example of ordering fractions with like denominators.

**Example 2:**  Order from least to greatest: $\frac{4}{5}$, $\frac{2}{5}$, $\frac{7}{5}$, $\frac{3}{5}$

| Fraction | Order |
|----------|-------|
| $\frac{4}{5}$ | ③ |
| $\frac{2}{5}$ | ① |
| $\frac{7}{5}$ | ④ |
| $\frac{3}{5}$ | ② |

**Solution:**     $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{7}{5}$

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**Example 3:**   It takes Jack three-fifths of an hour to complete his math homework, five-sixths of an hour to complete his reading homework, and two-thirds of an hour to complete his science homework. Order the time spent to complete Jack's homework by subject from least to greatest.

**Analysis:**   These fractions have *unlike denominators*. We will use the <u>least common denominator</u> (LCD) to write these fractions as equivalents fractions with *like denominators*, and then compare them two at a time.

The LCD of $\frac{3}{5}$, $\frac{5}{6}$ and $\frac{2}{3}$ is 30.

| | | | |
|---|---|---|---|
| **Math:** | $\frac{3}{5} = \frac{n}{30}$ | $\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$ | |
| **Reading:** | $\frac{5}{6} = \frac{n}{30}$ | $\frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}$ | |
| **Science:** | $\frac{2}{3} = \frac{n}{30}$ | $\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30}$ | |

| | | | |
|---|---|---|---|
| **Math:** | $\frac{3}{5} = \frac{n}{30}$ | $\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$ | ① |
| **Reading:** | $\frac{5}{6} = \frac{n}{30}$ | $\frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}$ | ③ |
| **Science:** | $\frac{2}{3} = \frac{n}{30}$ | $\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30}$ | ② |

**Solution:**     Ordering the time spent on Jack's homework from least to greatest, we get: **Math, Science and Reading**.

**To order fractions with unlike denominators, use the LCD to write them as equivalent fractions with like denominators. Then compare two fractions at a time. It is helpful to write a number in a circle next to each fraction to compare them more easily.**

**Converting Fractions to Mixed Numbers**

$$2\frac{3}{4} = \frac{(4x2)+3}{4} = \frac{11}{4}$$

# 5<sup>th</sup> Grade Math Unit 4 CCGPS Standards:

**Adding and Subtracting Fraction w/ Like Denominators**

Let's cut up a hexagon into 6 pieces:

Each piece ⬛ is $\frac{1}{6}$ of the hexagon. Right?

And ⬛⬛⬛⬛ is $\frac{4}{6}$ of the hexagon.

So, what if we wanted to add

$$\frac{1}{6} + \frac{4}{6} ?$$

Hmm... that would be

▲ + ▲▲▲▲

Count them up

= ▲ ▲ ▲ ▲ ▲ = $\frac{5}{6}$

1  2  3  4  5

$$\text{So } \frac{1}{6} + \frac{4}{6} = \frac{5}{6}$$

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### Adding and Subtracting Fractions w/ Unlike Denominators

This is a bit tricky, but you'll think it's easy once you get used to it!

Let's try this:

$$\frac{1}{2} + \frac{1}{3}$$

The main rule of this game is that we can't do anything until the denominators are the same!

We need to find something called the least common denominator (LCD)... It's really just the LCM of our denominators, 2 and 3.

The LCM of 2 and 3 is 6. So, our LCD 6.

We need to make this our new denominator...

Change the $\frac{1}{2}$ :

$$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

Change the $\frac{1}{3}$ :

$$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

$$\text{So } \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$$

Now we can do it!

$$\frac{3+2}{6} = \frac{5}{6}$$

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Multiplying Fractions

*To multiply a fraction and a whole number, multiply the numerator of the fraction by the whole number. The denominator stays the same.*

For example:

$$\frac{1}{3} \times 2 = \frac{1 \times 2}{3} = \frac{2}{3}$$

$$\frac{3}{4} \times 3 = \frac{3 \times 3}{4} = \frac{9}{4}$$

$$\frac{2}{7} \times 5 = \frac{2 \times 5}{7} = \frac{10}{7}$$

This rule will work anytime you multiply a fraction by a whole number.

Using this rule, answer the following question. ***A recipe calls for 2/3 cup of sugar, but you want to make 3 times the recipe. Describe in two to three sentences how you would figure out how much sugar you would need.***

***Harcourt School  Math Model: "Multiplying Fractions"*** video lesson:
http://www.harcourtschool.com/activity/math_models2/English/04/gr5ch18_1.html

Video #2:  http://www.harcourtschool.com/activity/show_me/e631.htm

***To multiply mixed numbers*** by any other number (fraction, whole number or another mixed number) you must first convert the mixed number to an improper fraction.
To convert a mixed number to an improper fraction, you multiply the denominator by the whole number and then add the numerator to get your new numerator. The denominator stays the same. This is just the opposite of what we did in unit 10.

EXAMPLES:

$$5\frac{1}{8} = \frac{8 \times 5 + 1}{8} = \frac{41}{8}$$

$$2\frac{3}{5} = \frac{5 \times 2 + 3}{5} = \frac{13}{5}$$

$$4\frac{1}{3} = \frac{3 \times 4 + 1}{3} = \frac{13}{3}$$

***Multiplying Mixed Numbers Using Pictures***: Common Core Aligned Grade 5 Video Lesson

http://learnzillion.com/lessons/129-multiply-mixed-numbers-using-pictures

**Dividing Fractions**

**There's a really cool trick for these...**

<div align="center">**FLIP and MULTIPLY!**</div>

**Check it out:**

$$\frac{1}{3} \div \frac{4}{5}$$

flip the second fraction...
and multiply!

$$\frac{1}{3} \times \frac{5}{4}$$

**That's it -- then GO FOR IT!**

$$\frac{1}{3} \div \frac{4}{5} = \frac{1}{3} \times \frac{5}{4} = \frac{1 \times 5}{3 \times 4} = \frac{5}{12}$$

---

1                                                                    2

We know that___goes into 1 exactly five times. But how many times does___go into 1?

5                                                                    5

Let's think with pictures.___goes into___two times, and then we have 1/5 left over.

2          1

How many times does___fit into___? Or, how many times does___go into___?

5          5

That is like trying to fit a two-piece part into a hole that just fits one piece. Only 1/2 of the part fits. So, 2/5 goes into 1/5 half a time. All in all, 2/5 fits into *one* exactly 2 1/2 times.

2          1

We can write a division from this: 1 ÷ ___ = 2 ___

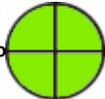5          2

---

Let's look at another similar example.

1                                                                    3
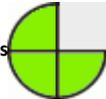
We know that___goes into 1 exactly four times. But how many times does___go into 1?

4                                                                    4

Let's look at pictures. [fraction circle 3/4] goes into [fraction circle] once, and we have 1/4 left over.

3        1

How many times does ___ fit into ___? Or, how many times does [fraction circle 3/4] go into [quarter piece]?
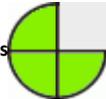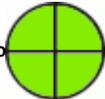
4        4

We have a three-piece (3/4) part trying to fit into one piece (1/4), so the part fits 1/3 of the way.

3        1

Putting it all together, 3/4 fits into one 1 1/3 times. As a division: 1 ÷ ___ = 1 ___

4        3

*How many times does one thing fit into another?* From this situation, you can always write a division, and think: "How many times does the *divisor* go into the *dividend*?"

1

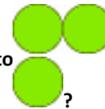How many times does ___ go into 2?

4

How many times does [quarter piece] go into [two circles]?

1

8 times. We write a division: 2 ÷ ___ = 8

4

1

How many times does ___ go into 3?

2

How many times does [half piece] go into [three circles]?

1

6 times. We write a division: 3 ÷ ___ = 6

2

Divide by Fractions w/ Models:  http://www.ixl.com/math/grade-6/divide-by-fractions-with-models

Harcourt Math Model: Divide Fractions:  http://www.harcourtschool.com/activity/math_models2/English/04/gr5ch19_1.html